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Translated by R.C.G.

PMM U.S.S.R.,Vol.48,No.1,pp.92-96,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

A CLASS OF EXACT SOLUTIONS WITH A UNIFORM DEFORMATION IN GAS DYNAMICS*

S.A. POSLAVSKII and I.S. SHIKIN

A new exact solution is obtained describing the motion of a rotating gas ellipsoid in which the ratio of the semiaxes remains constant (with an adiabatic index of 5/3). The structure of this solution is in a certain sense similar to the structure of the solutions for an ellipsoid of a uniform ideal incompressible selfgravitating liquid, obtained in papers on the theory of equilibrium figures (see 1, 2/).

The adiabatic motions of an ideal gas with a non-uniform deformation were first studied by Sedov /3, 4/, who obtained an exact non-stationary solution in the uniform case. Ovsyannikov /5/ showed that in a more general formulation the problem reduces to a set of nine second-order ordinary differential equations. This system allows seven first integrals connected with the conservation of energy and momentum of the gas cloud and "freezing in" of the vortex /5, 6/. An eighth integral was obtained in /7/ with an adiabatic index of 5/3, and an exact solution of the problem of the dispersion of a non-rotating gas ellipsoid of rotation in a vacuum was found. The problem of the motion of a rotating spheroid was considered in /8/. A qualitative investigation in the general case of motion with a uniform deformation of a triaxial gaseous ellipsoid was carried out in /9/.**

1. Solutions with non-uniform deformation are characterized by a linear depedence of the Euler coordinates on the Lagrangians

$$x_\alpha = M_{\alpha\beta}(t)\,\xi_\beta \tag{1.1}$$

Here and hence for the Greek subscripts take values from 1 to 3, and summation is carried out over the repeated indices.

For an ellipsoidal distribution of the density and pressure, the adiabatic motions of a gas are described by the equations

$$\ddot{M}_{\alpha\beta} = -\frac{\varepsilon}{D^{\gamma-1}}(M^{-1})_{\beta\alpha};\quad D = \det M,\quad \gamma = \frac{c_p}{c_v},\quad \varepsilon = \text{const} \tag{1.2}$$

The density and pressure are given by the equations

$$\rho = \frac{\rho_0(\sigma)}{D},\quad p = \frac{p_0(\sigma)}{D^\gamma};\quad p_0(\sigma) = p_0(0) + \int_0^\sigma \rho_0(\lambda)\,d\lambda,\qquad \sigma = \frac{\varepsilon}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2) \tag{1.3}$$

where $\rho_0(\sigma)$ is an arbitrary function. For a finite ellipsoid with boundaries $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$ the function $\rho_0(\sigma)$ vanishes outside it.

If $\varepsilon < 0$, the pressure falls with distance from the centre of the ellipsoid; if $\varepsilon > 0$, it increases. The first case may correspond, for example, to the motion of a gas cloud in a vacuum, and the second may correspond to the motion of an ellipsoid acted upon by an external pressure.

The matrix M can be represented in the form

$$M = Q_1 A Q_2 \tag{1.4}$$

*Prikl.Matem.Mekhan.,48,1,137-142,1984

**A single parametric family of exact solutions for a gas spheroid with a constant ratio of the semiaxes was obtained by Bogoyavlenskii in a paper entitled "The oscillatory expansion of a gas cloud in a vacuum", Preprint of the Institute of Theoretical Physics Academy of Sciences of the USSR, Chernogolovka, 1975.

where $A = \| A_{\alpha\beta} \|$ is a diagonal matrix, and $Q_1 = \| Q_{\alpha\beta}^{(1)} \|, Q_2 = \| Q_{\alpha\beta}^{(2)} \|$ are orthogonal matrices. The components $A_{\alpha\alpha}$ are the lengths of the principle semiaxes of the gas ellipsoid, Q_1 defines the orientation of these axes with respect to a fixed Euler system of coordinates, and the matrix Q_2 represents the rotation of the principle axes of the ellipsoid in a space of Lagrangian coordinates. Below we will consider the solutions for which

$$A_{\alpha\beta} = \lambda_\alpha d(t)\, \delta_{\alpha\beta} \tag{1.5}$$

where λ_α $(\alpha = 1, 2, 3)$ are positive constants, $\delta_{\alpha\beta}$ is the Kronecker delta, and there is no summation over α. When condition (1.5) is satisfied the ratio of the semiaxes of the gas ellipsoid does not change with time.

We will further assume that the adiabatic index $\gamma = {}^5/_3$. The integrals of (1.2), obtained in /5—7/, can be written in the form

$$e_{\alpha\beta\gamma} \dot{M}_{\gamma\sigma} M_{\beta\sigma} = J_\alpha, \quad e_{\alpha\beta\gamma} \dot{M}_{\sigma\gamma} M_{\sigma\beta} = K_\alpha \tag{1.6}$$

$$\frac{1}{2} \sum_{\alpha, \beta} \dot{M}_{\alpha\beta}^2 - \frac{3\varepsilon}{2} D^{-2/3} = E, \quad \sum_{\alpha, \beta} M_{\alpha\beta}^2 = 2Et^2 + C_1 t + C_2 \tag{1.7}$$

Here $e_{\alpha\beta\gamma}$ are the components of a perfectly antisymmetric unit pseudo-tensor $(e_{123} = 1)$; J_α, K_α $(\alpha = 1, 2, 3)$, E, C_1, C_2 are constants. The integrals (1.6) and (1.7) in general are insufficient to integrate (1.2), but when condition (1.5) is satisfied its solution can be written in quadratures.

2. It follows from (1.1) that

$$v_\alpha = \dot{x}_\alpha = \dot{M}_{\alpha\beta} (M^{-1})_{\beta\gamma} x_\gamma \tag{2.1}$$

We will introduce a fixed Cartesian system of coordinates X_1, X_2, and X_3, the axes of which are directed along the principal axes of the ellipsoid. We will fix the instant of time t^* and choose the Euler and Lagrangian coordinates so that their axes at this instant coincide with the corresponding axes of the movable system

$$x_\alpha(t^*) = X_\alpha(t^*); \quad \xi_\alpha = X_\alpha(t^*) / (\lambda_\alpha d(t^*)) \tag{2.2}$$

Then $Q_1(t^*)$ and $Q_2(t^*)$ will be unit matrices, and (2.1), taking (1.4) and (1.5), can be rewritten in the form /2/

$$v_\alpha = e_{\alpha\beta\gamma} X_\gamma \left(\frac{\lambda_\alpha^2}{\lambda_\alpha^2 + \lambda_\gamma^2} \zeta_\beta + \Omega_\beta \right) + \frac{\dot{d}}{d} X_\alpha \tag{2.3}$$

$$\zeta_\alpha(t^*) = -e_{\alpha\beta\gamma} \dot{Q}_{\beta\gamma}^{(2)}(t^*) \frac{\lambda_\beta^2 + \lambda_\gamma^2}{2\lambda_\beta\lambda_\gamma}, \quad \Omega_\alpha(t^*) = -\frac{1}{2} e_{\alpha\beta\gamma} \dot{Q}_{\beta\gamma}^{(1)}(t^*)$$

(in the frst formula there is no summation over α). Here $\boldsymbol{\Omega}$ is the vector of the instantaneous angular velocity of rotation of the system of principal axes of the ellipsoid in fixed space, and $\boldsymbol{\zeta}$ is the vector of the vorticity of the motion of the gas with respect to this system.

Using (1.4)—(1.6) we obtain

$$\dot{Q}_{\alpha\beta}^{(1)}(t^*) = -\frac{2\lambda_\alpha\lambda_\beta K_\gamma + (\lambda_\alpha^2 + \lambda_\beta^2) J_\gamma}{(\lambda_\alpha^2 - \lambda_\beta^2)^2 d^2} \tag{2.4}$$

$$\dot{Q}_{\alpha\beta}^{(2)}(t^*) = \frac{(\lambda_\alpha^2 + \lambda_\beta^2) K_\gamma + 2\lambda_\alpha\lambda_\beta J_\gamma}{(\lambda_\alpha^2 - \lambda_\beta^2)^2 d^2}$$

(α, β, γ form a cyclic permutation from 1, 2 and 3).

We will calculate the derivatives $d'K_\alpha/dt$, $d'J_\alpha/dt$ connected with the change in the choice of coordinates (2.2) with time. It can be seen from (1.1) and (1.6) that K_α are the components of the constant axial vector in the space of Lagrangian coordinates, while J_α are the components of the constant axial vector in Euler coordinates. The system of principal axes of the ellipsoid at the instant t^* is rotated in fixed space with instantaneous angular velocity $-e_{\alpha\beta\gamma}\dot{Q}_{\beta\gamma}^{(1)}(t^*)/2$, while in the space of Lagrangian coordinates it is rotated with instantaneous angular velocity $e_{\alpha\beta\gamma}\dot{Q}_{\beta\gamma}^{(2)}(t^*)/2$. Hence,

$$d'K_\alpha/dt = \dot{Q}_{\beta\alpha}^{(2)}(t^*) K_\beta, \quad d'J_\alpha/dt = \dot{Q}_{\alpha\beta}^{(1)}(t^*) J_\beta$$

We will introduce the following notation:

$$G_\alpha = -\frac{K_\alpha + J_\alpha}{2(\lambda_\beta - \lambda_\gamma)^2}, \quad H_\alpha = \frac{K_\alpha - J_\alpha}{2(\lambda_\beta + \lambda_\gamma)^2} \tag{2.5}$$

(α, β, γ form a cyclic permuation from 1, 2 and 3).

Then, for the quantitites G_α and H_α we will have the following system of equations:

$$\frac{d'G_\alpha}{d\tau} = e_{\alpha\beta\gamma}\frac{G_\beta G_\gamma(\lambda_\beta+\lambda_\gamma-2\lambda_\alpha)-H_\beta H_\gamma(\lambda_\beta+\lambda_\gamma+2\lambda_\alpha)}{2(\lambda_\beta-\lambda_\gamma)} \qquad (2.6)$$
$$\frac{d'H_\alpha}{d\tau} = e_{\alpha\beta\gamma}\frac{H_\beta G_\gamma(\lambda_\beta-\lambda_\gamma-2\lambda_\alpha)-H_\gamma G_\beta(\lambda_\beta-\lambda_\gamma+2\lambda_\alpha)}{2(\lambda_\beta+\lambda_\gamma)}$$

($d\tau = dt/d^2$ and there is no summation over α).

For this choice of coordinates (2.2), system (1.2), taking (1.5) into account, can re-written in the form ($\gamma = {}^5/_3$)

$$Q_1^{\cdot\cdot}A + A^{\cdot\cdot} + AQ_2^{\cdot\cdot} + 2(Q_1^{\cdot}A^{\cdot} + Q_1^{\cdot}AQ_2^{\cdot} + A^{\cdot}Q_2^{\cdot}) = -\varepsilon A^{-1}/D^{2/3} \qquad (2.7)$$

Since $Q_{\alpha\beta}^{(1)}(t^*) = Q_{\alpha\beta}^{(2)}(t^*) = \delta_{\alpha\beta}$, and for any orthogonal matrix Q the product $Q^{\cdot}Q'$ (Q' is the transposed matrix of Q), and of course, $(Q^{\cdot}Q')^{\cdot} = Q^{\cdot\cdot}Q' + Q^{\cdot}Q^{\cdot\cdot}$ are antisymmetric matrices, the following equation holds:

$$Q_{\alpha\alpha}^{\cdot\cdot(n)}(t^*) = -\sum_\beta Q_{\alpha\beta}^{\cdot(n)}(t^*)\,Q_{\alpha\beta}^{\cdot(n)}(t^*) \quad (\alpha = 1, 2, 3;\ n = 1, 2) \qquad (2.8)$$

Using (1.5), (2.3), and (2.8), the diagonal components of the matrix equation (2.7) can be written in the form

$$d^3d^{\cdot\cdot} = \left[\Omega_3^2 + \Omega_2^2 + \frac{\lambda_1^2\lambda_2^2}{(\lambda_1^2+\lambda_2^2)^2}\zeta_3^2 + \frac{\lambda_1^2\lambda_3^2}{(\lambda_1^2+\lambda_3^2)^2}\zeta_2^2 + \frac{2\lambda_2^2}{\lambda_1^2+\lambda_2^2}\zeta_3\Omega_3 + \frac{2\lambda_3^2}{\lambda_1^2-\lambda_3^2}\zeta_2\Omega_2\right]d^4 - \frac{\varepsilon}{\lambda_1^{8/3}(\lambda_2\lambda_3)^{2/3}} \qquad (2.9)$$

The remaining two equations are obtained by cyclic permutation of the indices.

It follows from the last integral in (1.8) and condition (1.5) that

$$d^3d^{\cdot\cdot} = (2EC_2 - C_1^2/4)/(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) = C_3 \qquad (2.10)$$

Using (2.3)–(2.5) and (2.9) the last equation can be written in the form

$$(\lambda_\alpha+\lambda_\beta)H_\gamma^2 + (\lambda_\alpha-\lambda_\beta)G_\gamma^2 + (\lambda_\alpha+\lambda_\gamma)H_\beta^2 + (\lambda_\alpha-\lambda_\gamma)G_\beta^2 = \frac{\varepsilon}{2\lambda_\alpha^{5/3}(\lambda_\beta\lambda_\gamma)^{2/3}} + \frac{C_3\lambda_\alpha}{2} \qquad (2.11)$$

(α, β, γ form a cyclic permutation from 1, 2 and 3).

Riemann showed that the solutions of a system of the form (2.6) satisfying relations of the form (2.11), can only be singular points, for which the right sides of (2.6) vanish. Moreover, if not all λ_β ($\beta = 1, 2, 3$) are equal to one another, an index α exists such that $G_\alpha = H_\alpha \equiv 0$. Suppose, to be specific, that $\alpha = 1$. Then $\zeta_1 = \Omega_1 \equiv 0$. The vectors ζ and Ω lie in one of the principal planes of the ellipsoid. We will put $\zeta_\alpha^\circ = d^2\zeta_\alpha$, $\Omega_\alpha^\circ = d^2\Omega_\alpha$. It follows from (2.3)–(2.5) and the fact that the right sides of (2.6) are equal to zero, that the components of the vectors ζ°, Ω° in X_α coordinates do not change during motion.

3. Suppose $\zeta_2 = \Omega_2 \equiv 0$, i.e. the gas ellipsoid rotates around its own axis. The case when $\lambda_1 = \lambda_2$ was considered in /8/. Hence, we will assume that $\lambda_1 \neq \lambda_2$.

Using (2.9) we obtain

$$\Omega_3^\circ + \frac{\lambda_1\lambda_2}{\lambda_1^2+\lambda_2^2}\zeta_3^\circ = \pm R_-^{1/2}, \quad \Omega_3^\circ - \frac{\lambda_1\lambda_2}{\lambda_1^2+\lambda_2^2}\zeta_3^\circ = \pm R_+^{1/2} \qquad (3.1)$$
$$R_\pm = -\frac{\varepsilon}{(\lambda_1\lambda_2)^{5/3}\lambda_3^{2/3}}\left(\frac{\lambda_1\lambda_2}{\lambda_3^2} \pm 1\right)$$
$$d^3d^{\cdot\cdot} = -\frac{\varepsilon}{(\lambda_1\lambda_2)^{2/3}\lambda_3^{8/3}} \qquad (3.2)$$

The qantities R_+, R_- must be non-negative. Hence, in the solutions of the type considered $\lambda_1\lambda_2 \geqslant \lambda_3^2$ and $\varepsilon < 0$, i.e. the pressure falls with distance from the centre of the ellipsoid. Integrating (3.2) we obtain

$$d = \left[-\frac{\varepsilon(t-t_0)^2}{(\lambda_1\lambda_2)^{2/3}\lambda_3^{8/3}d_0^2} + d_0^2\right]^{1/2}; \quad d_0 = d(t_0), \quad d^{\cdot}(t_0) = 0 \qquad (3.3)$$

The general solution of (1.2) in the case considered has the form

$$M = S_1Q_1AQ_2S_2 \qquad (3.4)$$
$$Q_1 = \begin{Vmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{Vmatrix}, \quad Q_2 = \begin{Vmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{Vmatrix}$$

$$\varphi=\int\frac{\Omega_3^\circ}{d^2}\,dt,\quad \psi=\frac{\lambda_1\lambda_2}{\lambda_1^2+\lambda_2^2}\int\frac{\zeta_3^\circ}{d^2}\,dt$$

Here S_1 and S_2 are arbitrary constant orthogonal matrices, A has the form (1.5), d is found from (3.1), and ζ_3° and Ω_3° are found from (3.1).

4. Suppose now that $\Omega_2\neq 0,\ \Omega_3\neq 0$. Without loss of generality we will assume that $\lambda_1=1$. The fact that the right sides of (2.6) are equal to zero can be written in the form /2/

$$\beta_1+\gamma_1-\beta_1\gamma_1=\frac{\lambda_2^2+\lambda_3^2}{2},\quad \beta_1-\gamma_1=\frac{\lambda_3^2-\lambda_2^2}{2} \tag{4.1}$$

$$\left(\beta_1=-\frac{\lambda_3^2}{1+\lambda_3^2}\frac{\zeta_2^\circ}{\Omega_2^\circ},\quad \gamma_1=-\frac{\lambda_2^2}{1+\lambda_2^2}\frac{\zeta_3^\circ}{\Omega_3^\circ}\right)$$

Hence

$$\frac{\zeta_2^\circ}{\Omega_2^\circ}=-\frac{1+\lambda_3^2}{4\lambda_3^2}(4-\lambda_2^2+\lambda_3^2\pm R^{1/2}), \tag{4.2}$$

$$\frac{\zeta_3^\circ}{\Omega_3^\circ}=-\frac{1+\lambda_2^2}{4\lambda_2^2}(4-\lambda_3^2+\lambda_2^2\pm R^{1/2})$$

$$R=[4-(\lambda_2+\lambda_3)^2]\,[4-(\lambda_2-\lambda_3)^2]$$

On the other hand, from (2.9), using (4.1), we obtain

$$(\Omega_2^\circ)^2\beta_1=-\frac{\lambda_3^2}{1+\lambda_3^2}\zeta_2^\circ\Omega_2^\circ=\frac{2}{3}\frac{\varepsilon}{\lambda_2^{8/3}\lambda_3^{2/3}}\frac{(1-\lambda_2^2)(\lambda_3^2-4)}{\lambda_3^2-\lambda_2^2} \tag{4.3}$$

$$(\Omega_3^\circ)^2\gamma_1=-\frac{\lambda_2^2}{1+\lambda_2^2}\zeta_3^\circ\Omega_3^\circ=\frac{2}{3}\frac{\varepsilon}{\lambda_2^{2/3}\lambda_3^{8/3}}\frac{(1-\lambda_3^2)(\lambda_2^2-4)}{\lambda_2^2-\lambda_3^2}$$

When solving the algebraic equations (4.2) and (4.3) we can express $\zeta_\alpha^\circ,\ \Omega_\alpha^\circ\ (\alpha=2,3)$ in terms of $\varepsilon,\ \lambda_2,\ \lambda_3$.

Using (2.9), (4.1), and (4.3), we will calculate the value of the constant C_3 on the right side of (2.10). We obtain

$$C_3=\frac{\varepsilon}{3(\lambda_2\lambda_3)^{8/3}}(\lambda_2^2+\lambda_3^2-\lambda_2^2\lambda_3^2-4) \tag{4.4}$$

After integrating (2.10) we obtain $d(t)$, similar to (3.3).

The general solution of (1.2) in this case has the form (3.4), but now

$$Q_1=\begin{Vmatrix}\cos\varphi & -\sin\varphi & 0\\ \sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{Vmatrix}$$

$$Q_2=\begin{Vmatrix}1 & 0 & 0\\ 0 & \cos\beta & \sin\beta\\ 0 & -\sin\beta & \cos\beta\end{Vmatrix}\cdot\begin{Vmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{Vmatrix}$$

$$\varphi=\int\frac{\Omega^\circ}{d^2}\,dt,\quad \psi=\int\frac{\zeta^\circ}{d^2}\,dt$$

$$\Omega^\circ=[(\Omega_2^\circ)^2+(\Omega_3^\circ)^2]^{1/2},\quad \zeta^\circ=\left[\left(\frac{\lambda_3}{1+\lambda_3^2}\zeta_2^\circ\right)^2+\left(\frac{\lambda_2}{1+\lambda_2^2}\zeta_3^\circ\right)^2\right]^{1/2}$$

$$\alpha=\operatorname{arctg}\frac{\Omega_2^\circ}{\Omega_3^\circ},\quad \beta=\operatorname{arctg}\left(\frac{\lambda_3(1+\lambda_2^2)\zeta_2^\circ}{\lambda_2(1+\lambda_3^2)\zeta_3^\circ}\right)$$

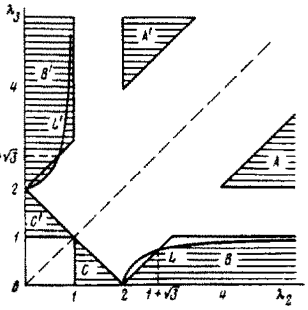
It should be noted that if the matrix $M(t)$ is the solution of (1.2), the transposed matrix $M'(t)$ will also be a solution.

The set of algebraic equations (4.2) and (4.3) is not solvable for possible values of $\varepsilon,\ \lambda_2,\ \lambda_3$. Without loss of generality we can confine ourselves to considering the case when $\lambda_2>\lambda_3$. It follows from (4.1) and the fact that the expressions under the root signs in (4.2) are non-negative that there are two possibilities

1) $\gamma_1>0,\ \beta_1<0,\ \lambda_2-\lambda_3\geqslant 2$
2) $\gamma_1>0,\ \beta_1>0,\ \lambda_2+\lambda_3\leqslant 2$

Assuming that the signs on the right and left sides of (4.3) must be same, we obtain as a result three regions of admissible values of $\varepsilon,\ \lambda_2,\ \lambda_3$ (see the figure)

a) $\lambda_2-\lambda_3\geqslant 2;\ \lambda_2\geqslant\lambda_3\geqslant 2;\ \varepsilon<0$ (region A)
b) $\lambda_2-\lambda_3\geqslant 2;\ \lambda_2\geqslant 1\geqslant\lambda_3;\ \varepsilon>0$ (region B)
c) $\lambda_2+\lambda_3\leqslant 2;\ \lambda_2\geqslant 1\geqslant\lambda_3;\ \varepsilon<0$ (region C)



In the solutions corresponding to cases a), and c), the pressure falls with distance from the centre of the gas ellipsoid. These solutions do not have any singularities, since $d^3d^{\cdot\cdot} = C_3 > 0$. In case b) the cloud moves due to the action of the external pressure, which varies with time as given by (1.3).

As can be seen from (4.4), C_3 vanishes along the line L, specified by the equation

$$\lambda_2^2 + \lambda_3^2 - \lambda_2^2\lambda_3^2 - 4 = 0$$

and splits the region B into two subregions (see the figure). For a point lying in region B above the line L, $C_3 < 0$, i.e. a singularity ($d = 0$) must necessarily occur in the solutions corresponding to it, namely, a state in which the volume of the cloud vanishes, while the density and pressure become infinite. For points lying below L, $C_3 > 0$, i.e. d does not vanish; the rotation and internal vorticity of the gas cloud prevents its collapse.

A complete picture in the (λ_2, λ_3) plane is obtained after symmetric reflection of regions A, B, and C in the straight line $\lambda_2 = \lambda_3$, and of the line L into regions A', B', and C' and the line L' respectively.

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Translated by R.C.G.

PMM U.S.S.R., Vol.48, No.1, pp.96-98, 1984
Printed in Great Britain
0021-8928/84 $10.00+0.00

RELATIVISTIC PRANDTL- MEYER FLOW*

N.I. KOLOSNITSYN and K.P. STANYUKOVICH

The exact solution of the equations of relativistic gas dynamics describing plane steady-state flow, depending only on the angular variable, is investigated. The well-known Prandtl-Meyer solution is obtained in the non-relativisitic limit.

The problem of constructing relativistic Prandtl-Meyer flow has been considered in /1—3/. A solution was obtained in /1/, by direct integration of the equations, describing the limiting case of ultrarelativistic flow. In /2, 3/, to obtain relativistic Prandtl-Meyer flow, the method of replacement of variables proposed in /4/ was used, by means of which the equations of relativistic hydrodynamic were reduced to Newtonian form for a certain auxiliary gas with a variable isentropy index. Using this

**Prikl.Matem.Mekhan.*, 48, 1, 143-145, 1984